Violent Relaxation
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This process happens at early stages of evolution of a system. The key feature is the explicit dependence of gravitational potential \( \psi(x,t) \) on time \( t \):

\[ \psi = \psi(x,t) \]

Let's try to integrate energy of a particle along its trajectory: \( E = \frac{v^2}{2} + \psi(x,t) \)

\[ \frac{dE}{dt} \bigg|_{\text{along trajectory}} = \frac{1}{2} \frac{dv^2}{dt} + \frac{d\psi}{dt} \]

but

\[ \frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} \frac{dx}{dt} + \frac{\partial \psi}{\partial t} \frac{dt}{dt} + \frac{\partial \psi}{\partial z} \frac{dz}{dt} = \frac{\partial \psi}{\partial t} + (\vec{v} \cdot \vec{a}) \psi \]

and

\[ \frac{1}{2} \frac{dv^2}{dt} = \vec{v} \cdot \vec{a} = - (\vec{v} \cdot \vec{a}) \psi \]  

Here we used eq. of motion:

\[ \frac{dv}{dt} = - \nabla \psi \]

Thus,

\[ \frac{dE}{dt} = \frac{\partial \psi}{\partial t} \]

\[ \Delta E \bigg|_{\text{along trajectory}} = \int_{t_0}^{t} dt \cdot \frac{\partial \psi}{\partial t} \]

We cannot take this integral, because we do not know how \( \psi \) changes with time. At early stages of evolution we expect that grav. potential changes significantly over short period of time. The shortest time is the dynamical time. Thus \( \Delta \psi \approx \psi \) for \( t \approx \text{dynamical} \)

Thus, our expectations:

\[ \Delta E \approx \int_{t_0}^{t} dt \frac{\partial \psi}{\partial t} \approx \Delta \psi \approx E \]

Toy model for violent relaxation:

\[ \circ \rightarrow \text{Particles, which come to the center after collapse, gain energy.} \]

\[ \circ \rightarrow \text{Particles, which cross the central region earlier, lose energy.} \]
Properties of violent relaxation

\[ t_{\text{viol.}} \sim t_{\text{dyn}} \]

\[ \Rightarrow \text{no dependency on particle mass:} \]
\[ \langle V_1^2 \rangle = \langle V_2^2 \rangle \text{ for } m_1 \neq m_2 \]

\[ \Rightarrow \text{Anisotropy is significantly reduced:} \]

\[ \Rightarrow \text{A fraction of particles may escape} \]

\[ \Rightarrow \text{Once the system reaches a quasi-static state,} \]
\[ \gamma = \gamma(x), \text{ violent relaxation shuts off.} \]
\[ \text{As such, it has a tendency to be incomplete.} \]
\[ \text{For example, initially anisotropic system} \]
\[ \text{will reduce anisotropy, but it will not become} \]
\[ \text{round.} \]

\[ \Rightarrow \text{Redistribution of energy between particles is} \]
\[ \text{typical for violent relaxation:} \]

![Energy distribution diagram](image)