Convection
Fluid Instabilities

1) Convection
2) Rayleigh-Taylor
3) Kelvin-Helmholtz
4) Thermal

Hydrostatics: \[ \text{grad } p = \rho g \]
\[ \text{g} = \text{grav. acceleration} \]

Is it stable?
Let's assume that all thermodynamical parameters depend only on one coordinate \( z \) (say, distance to the center of a star).

\[ p = p(z), \quad s = s(z), \quad V = V(z) \]
\[ s = \text{entropy}, \quad V = \text{specific volume} \]

Consider an element of fluid, which is displaced from \( z \) to \( z + dz \). The process is adiabatic - no energy exchange. The element is always in hydrodynamical equilibrium with surrounding fluid: its pressure is the same as the pressure of gas outside the element.

\[ V'(p', s') \quad \text{gas} \]
\[ p', s' \]

Stability condition: after the displacement, the density of the volume element is larger than the density of gas.

\[ V(p', s) < V(p', s') \]

Taylor expansion:
\[ s' - s = \frac{ds}{dz} \cdot dz \]

\[ V(p', s') = V(p', s) + \left( \frac{\partial V}{\partial s} \right)_p \frac{ds}{dz} \cdot dz \]

Thermodynamics:
\[ \left( \frac{\partial V}{\partial s} \right)_p = \rho \left( \frac{\partial V}{\partial T} \right)_p > 0 \Rightarrow \frac{ds}{dz} > 0 \]

Condition for absence of convection
For ideal gas this can be reduced to a condition for temperature gradient:

\[-\frac{dT}{dz} < \frac{g}{c_p}\]

**Rayleigh–Taylor instability:** heavy fluid atop of light one in presence of gravity g

**Example:** shock waves sweeps gas, once shock is exhausted, it leaves boundary a low density gas around the star with dense shell outside

Physically, R-T instability is the same as convective instability.

Convection will tend to reduce the entropy gradient and temperature gradient. If convection takes place:

\[\frac{dS}{dz} = 0\]

Convection can be laminar or turbulent (the transition is defined by Rayleigh number, a combination:

\[R = \frac{g \beta \Delta T}{\nu \kappa}\]

which includes: 
- temperature gradient, 
- typical length scale, 
- viscosity and thermal conductivity coefficients.

\[\frac{\Delta T}{\nu} = a \Delta \frac{\Delta T}{\nu}\]

\[\kappa = \kappa_{\text{thermal}}\]

\[\nu = \nu_{\text{kinetic}}\]